

# A Tour of Mathematics - Exploring the Borders

GEMS 105 - SPRING 2003

Tour Guide: Tim Pennings

Office: VWF 213

Office Hours: 10-11 daily

Welcome to GEMS 105, A Tour of Mathematics. My name is Tim Pennings - I will be your guide as we explore this large and influential country. On this seven week adventure tour, we will be visiting many of the main attractions of Mathematics including numbers, the infinite, analysis (differential and integral calculus), dynamical systems (including chaos and fractals), graph theory, geometry, and, in view of the recent election, voting.

You may have heard of some of these mathematical places - others may be entirely new to you. Obviously, we won't have time to visit any of them in their entirety. But in each place we will have time to see some of the most interesting sites and also take some adventure hikes. During our trip, you will also have opportunity to explore some topics on your own and report back to the others about them.

Maybe the most important and novel aspect of the tour will be our frequent visits to the borders of other countries such as philosophy, religion, politics, architecture, music, biology, art, cosmology, and meteorology. You will see how Mathematics helps us understand and more fully appreciate the richly interwoven world in which we live.

The goals for this tour of mathematics are for you to:

- Become acquainted with some of the great places in Mathematics;
- Appreciate the creativity and tenacity that led to the discoveries;
- Appreciate the utility and ubiquity of Mathematics in our daily lives;
- Appreciate some of the inner beauty of Mathematics;
- Learn how these places influence and affect our lives;
- Learn what these places have to tell us about the rest of our world;
- Engage in the discovery process yourself.

Since you are paying for this tour, I want to help ensure that you are getting your money's worth. We'll do that through i) daily quizzes, ii) reports and class participation, and iii) a comprehensive final (each component counting  $1/3$  of the final grade). The quizzes will provide continual feedback and motivation to keep up with the material. The reports (written and oral) and participation will hopefully keep everyone actively engaged in the tour and make it more enjoyable for us all. The comprehensive final is meant to keep all the topics ruminating in your mind throughout the entire tour so that connections and comparisons can be made as we go.

Our daily routine will typically be:

- Review problems and reports – 30 minutes
- Daily quiz – 15 minutes
- Go over quiz – 5 minutes
- Lecture on new material – 50 minutes

Daily assignments will include reading, working on QUESTIONS FOR FURTHER DISCOVERY, and preparing occasional reports for the class. The reading is filled with questions, and is meant to be done slowly and reflectively. Curl up in front of a cozy fire with a mug of hot chocolate and a Welsh Corgi on your lap as you read – and let your mind gently drift as you ponder - mentally exploring the treasures of Mathematics . . .

A good rule of thumb for out-of-class work is to study two hours outside of class for each hour in class. So assuming 4 hours of work between class periods, I would recommend spending  $1\frac{1}{2}$  hours reading reflectively, about 2 hours working on the QUESTIONS FOR FURTHER DISCOVERY, and  $1/2$  hour reviewing previous material. Then you will need no extra preparation for the final.

Our tentative itinerary is given below. If there is a place you'd like to visit which is not on the tour, adjustments can be made.

1/8 Numbers - Natural Numbers, Integers, Prime Numbers

1/13 Numbers - Rational, Real, and Related Topics

1/16 Tour guide gone. Watch "The Proof" and "Donald in Mathemagic Land".

1/20 The Infinite - Sets, Cardinality, Sizes of  $\mathcal{N}$ ,  $\mathcal{Q}$ , and  $\mathcal{R}$

1/22 The Infinite: How many sizes, and Related Ideas

1/27 Geometric Shapes - The Effects of Scaling

1/29 Geometry: Absolute Truth and Bastards in the Family

2/3 The Calculus of Slopes

2/5 The Calculus of Area

2/12 Graphs

2/17 Dynamical Systems: Chaos and Fractals

2/19 Voting Strategies

2/24 Sequences

2/26 Final. Go over it together.