

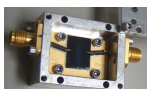


# Harmonic and Intermodulation Distortion in a Superconducting Microwave Resonator

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## A $Tl_2Ba_2CaCu_2O_{8-y}$ microstrip resonator

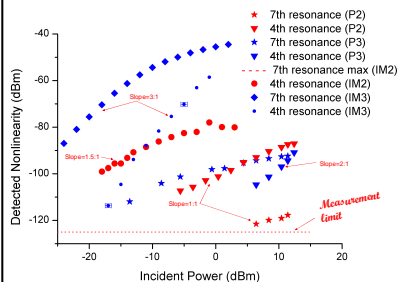
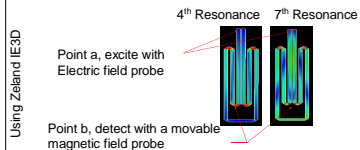


400 nm thick TBCCO film patterned on a 0.5 mm thick LAO substrate by Argon ion beam milling.

### Measured Resonances

n	Resonant Freq.	$Q_n$
2	1.5640 GHz	
3	2.2178 GHz	6,000
4	2.8700 GHz	11,000
5	3.5240 GHz	15,400
6	4.3722 GHz	2,700
7	4.5885 GHz	8,500

### Current distributions in the resonance



The 7th resonance of this structure exhibits very little time reversal symmetry breaking, when compared to the 4th resonance.

### References

Sheng-Chang Lee, "Measurement of Doping-Dependent Microwave Nonlinearity in High-Temperature Superconductors", PhD Dissertation, University of Maryland, 2004.

Sheng-Chang Lee, Matthew Sullivan, Gregory R. Ruzh, Steven Anlage, Sergio S. Pappas, B. Macon, and E. Cheung, "Doping-dependent nonlinear microwave and superconductivity in high- $T_c$  superconductors", Phys. Rev. B, 71, 040507 (2005).

S. K. Remillard, H.R. Y. and A. Abdalmonem, "Three-Tone Intermodulation Distortion Generated by Superconducting Baroque Filters", IEEE Transactions on Applied Superconductivity, 13, 3797 (2003).

S.K. Remillard, L.J. Slaughter, J.D. Hedge, T.A. Freeman, P.A. Smith, and T.W. Butler, "Generation of Intermodulation Products by Granular  $Tl_2CuO_4$  Thin Films", Proceedings of the SPIE Conf. High-Temp. Microwave Superconductors and Applications, 2009, San Diego, CA, USA, July 9-14, (1995).

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## Objective

To observe the conditions for, and seek explanations of, even and odd order nonlinearity

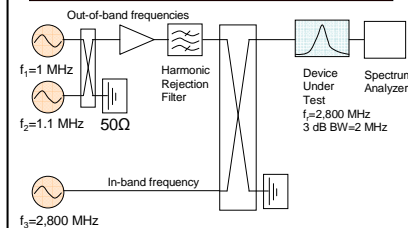
**Harmonic distortion** is the production of outputs at an integer multiple of the input frequency. For example,

Even order (P2, P4...):  $f_1$  input,  $2f_1, 4f_1, 6f_1, \dots$  output.  
Odd order (P3, P5...):  $f_1$  input,  $3f_1, 5f_1, 7f_1, \dots$  output.

**Intermodulation Distortion (IMD)** is the mixing of two or more signals in a nonlinear device. For example,

2<sup>nd</sup> order (IM2):  $f_1$  and  $f_2$  input,  $f_1+f_2$  output.  
3<sup>rd</sup> order (IM3):  $f_1$  and  $f_2$  input,  $2f_1-f_2$  output.

## Next Step: 3-Tone IMD



All nonlinearities are on-resonance and share the field distribution and coupling coefficient of the resonant mode.

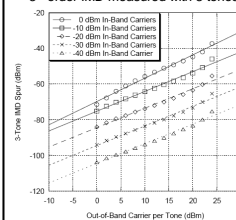
### Advantages of 3-tone IMD

1) All measured nonlinearities are on resonance, permitting the use of a single, easily measured, reflection coefficient,  $\Delta S_{11}$ , and correlation of nonlinearity current to circulating power.

3<sup>rd</sup> order is at  $f_3 \pm (f_2 - f_1)$   
2<sup>nd</sup> order is at  $f_3 \pm f_2$  and  $f_3 - f_1$

2) High power is out of band resulting in enhanced test system sensitivity and minimizing peak distortion.

### 3<sup>rd</sup> order IMD measured with 3 tones



The 3-tone IMD measured for an 8-pole, 7.5 MHz wide filter centered at 903.75 MHz fabricated from 400 nm thick  $Tl_2Ba_2CaCu_2O_x$  thin film on a Lanthanum Aluminate (LAO) substrate was measured at 75 Kelvin. The two out-of-band carriers were spaced 0.5 MHz apart. (Figure from Remillard, et al., IEEE Trans. Appl. Supercond. 13, no. 3, Sept. 2003.)

## Even versus Odd Order Nonlinear Distortion

Taylor expand the electric field, E, in a magnetic field  $H = H_0(\cos(2\pi f_1 t) + \cos(2\pi f_2 t))$  with 2 tones

$$E(H) = E(0) + \frac{dE}{dH} \Big|_{H=0} H + \frac{d^2E}{dH^2} \Big|_{H=0} \frac{H^2}{2!} + \frac{d^3E}{dH^3} \Big|_{H=0} \frac{H^3}{3!} + \dots$$

See Remillard (1995)

1<sup>st</sup> order term at  $f_1$  and  $f_2$

→ Preserves time reversal symmetry

3<sup>rd</sup> order terms at such frequencies as  $3f_1, 2f_2 \pm f_1$

→ Preserves time reversal symmetry

2<sup>nd</sup> order terms at such frequencies as  $2f_1, f_2 \pm f_1$

→ Breaks time reversal symmetry

Time reversal symmetry (TRS) is determined by constitutive relations

$$\vec{E}_{||} = -L(H) \frac{dH}{dt}, \text{ width} \quad \text{Where } L \text{ depends on } H = H_0 \sin(\omega t), \text{ e.g. } L \text{ is nonlinear.}$$

See Lee (2005)

$$L(H) = L_0 + \sum_m L(H^m)$$

Inductance per unit length

Even time reversal symmetry

E and dH/dt have even TRS. H has odd TRS.

• m even → TRS is preserved, and odd order harmonics occur.

• m odd → TRS is broken, and harmonics in E with  $\sin((m+1)\omega t)$ , where m+1 is even, occur.

## Acknowledgements



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