Simultaneous and Synchronous Measurement of Even and Odd Order Nonlinear Distortion Terms

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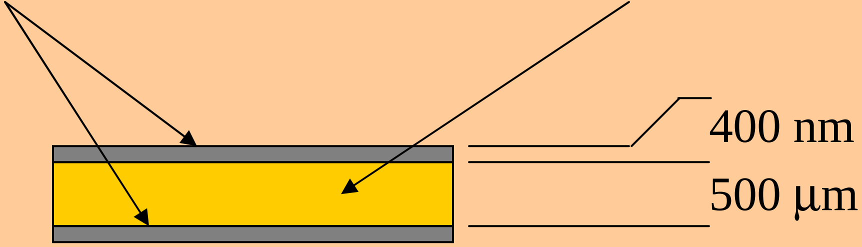
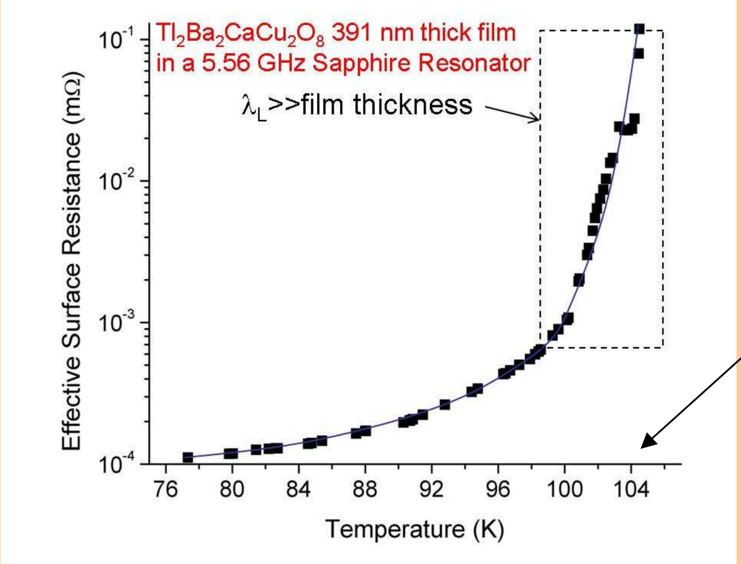
Motivations

- Robust passive intermodulation distortion (IMD) measurement
- Synchronous measurement of even and odd order distortion currents
- Detection of time reversal symmetry breaking (TRSB) in high temperature superconductor (HTS) circuits
- Added stipulation: this research must be undergraduate accessible.

Superconductors

$Tl_2Ba_2CaCu_2O_8$ thin films on $LaAlO_3$ substrate

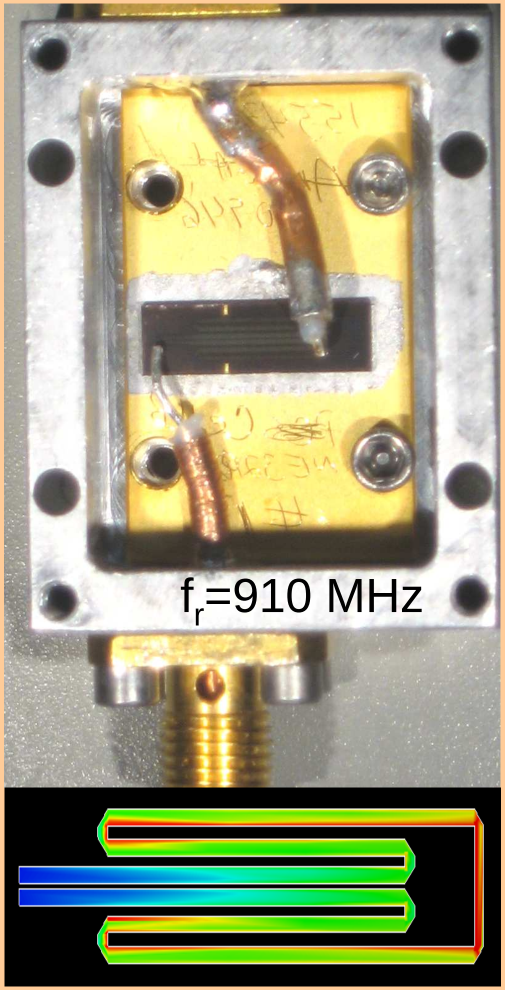




Critical temperature around 104 K

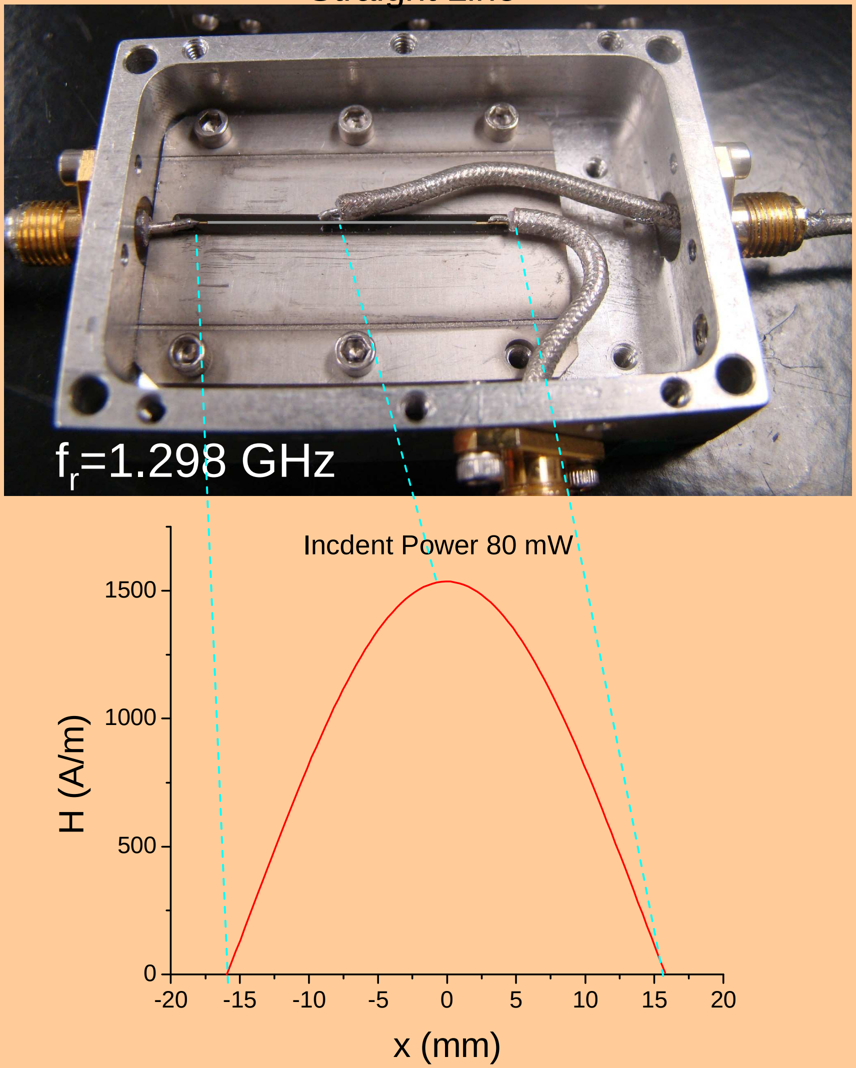
Two Superconducting Resonators

“Guitar”



Simulations done with IE3D, Zeland Software

“Straight Line”



Nonlinear Distortion in Superconductors

The inductance of a nonlinear transmission line:

Must be independent of direction of the current in order to obey *Time Reversal Symmetry.*

$$L(H) = L_o + \Delta L \cdot \left(\frac{H}{H_c}\right)^2$$

$$H(t) = H_o \sin(\omega t)$$

$$E = -L\frac{dH}{dt} \longrightarrow E\left[\cos(\omega t), \cos(3\omega t)\right]$$

H_c=the nonlinearity scaling field.

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Nonlinear Distortion in Superconductors

The inductance of a nonlinear transmission line:

Dependence of inductance on direction **breaks** time reversal symmetry.

$$L(H) = L_o + \Delta L' \cdot \left(\frac{H}{H_{TRSB}} \right) + \Delta L \cdot \left(\frac{H}{H_c} \right)^2$$

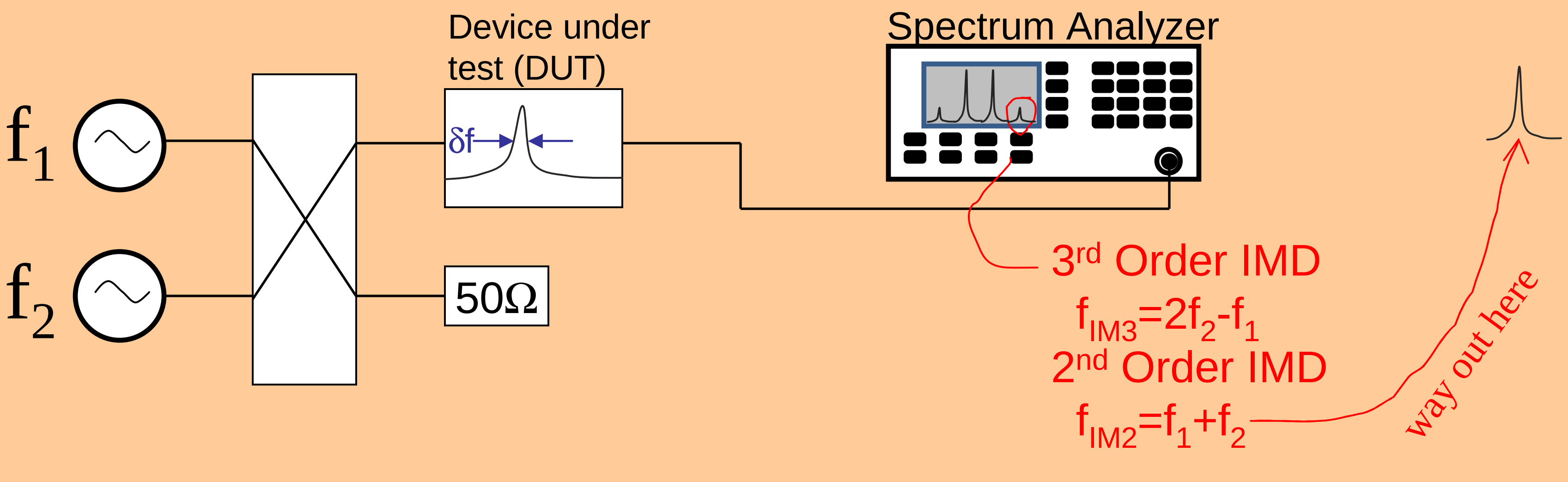
$$H(t) = H_o \sin(\omega t)$$

$$E = -L \frac{dH}{dt} \longrightarrow E\left[\cos(\omega t), \cos(2\omega t), \cos(3\omega t)\right]$$

H_{TRSB}=the scaling field for time reversal symmetry breaking.

Two tone IMD

“conventional method”



f_1

f_2

Device under test (DUT)

δf

50Ω

Spectrum Analyzer

3rd Order IMD

$f_{IM3}=2f_2-f_1$

2nd Order IMD

$f_{IM2}=f_1+f_2$

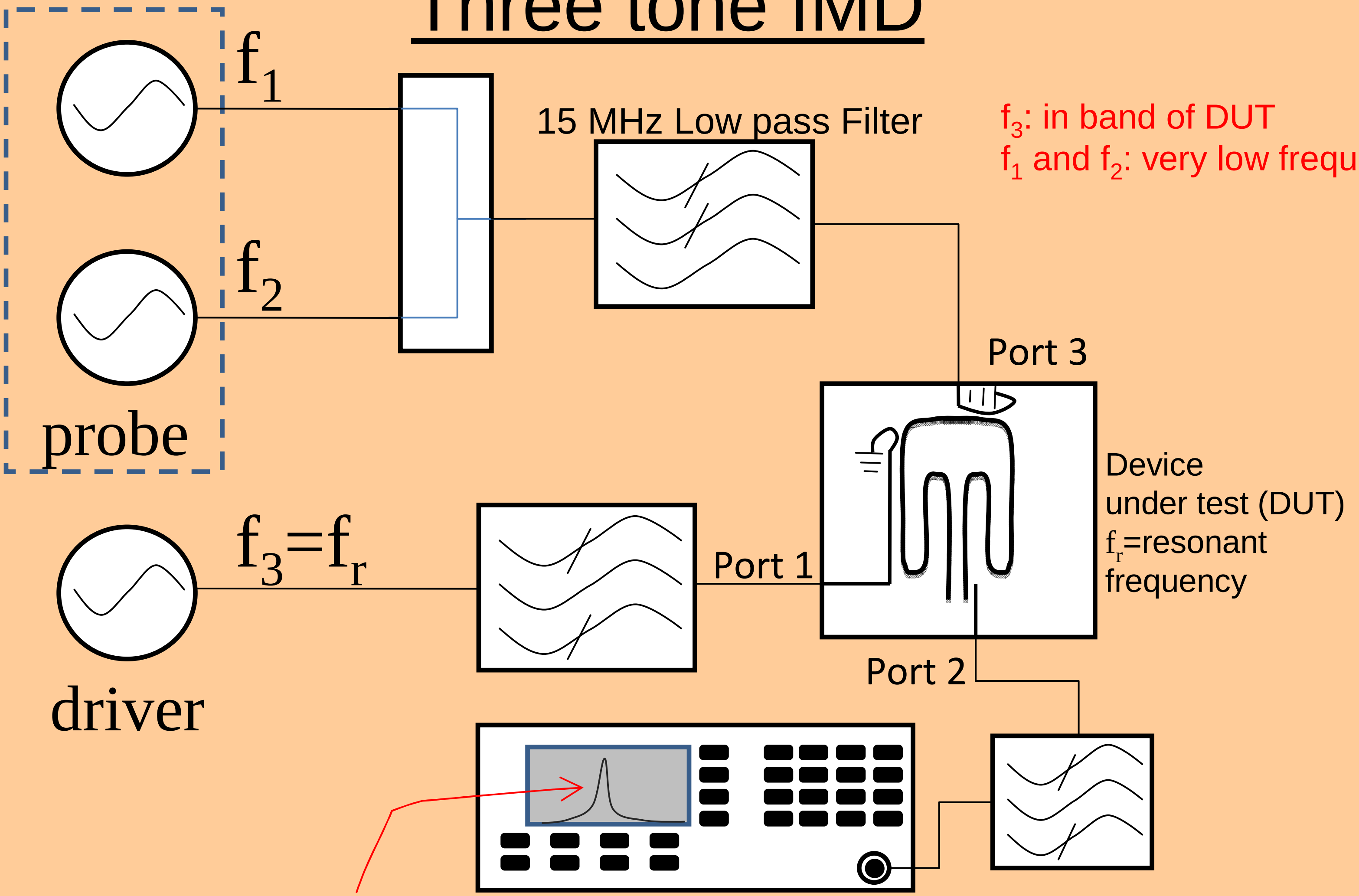
way out here

Limitations of the usual method

- 2nd order IMD is usually out of band of the DUT
- High power carriers can mix in the test set-up

  ➔ Not robust

Three tone IMD

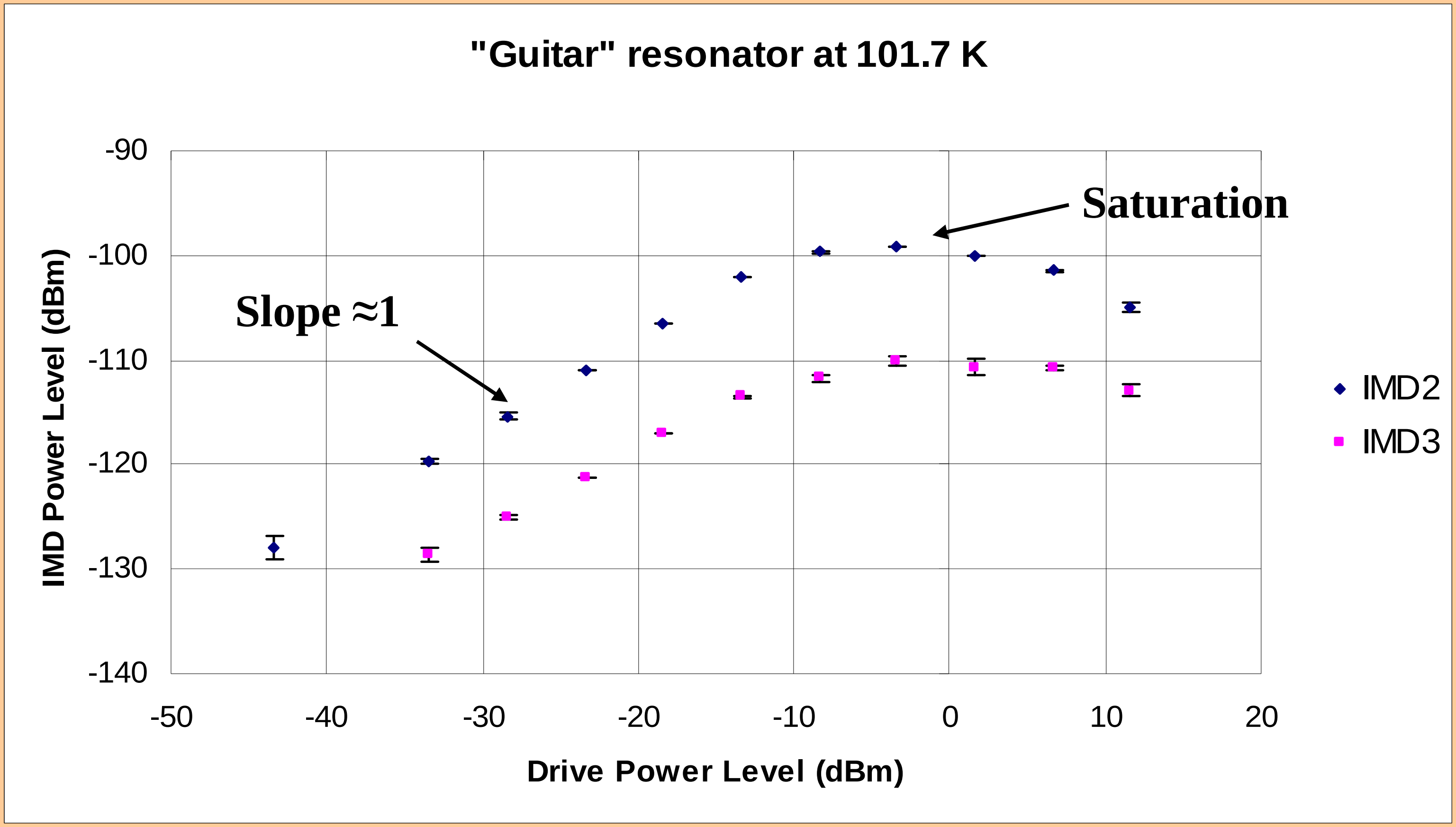


f_3: in band of DUT
f_1 and f_2: very low frequency

3rd Order IMD, $f_{IM3}=f_3+(f_2-f_1)$ or
2nd Order IMD, $f_{IM2}=f_3+f_1$ (technically a 2-tone measurement)

(Pease, at al,.Rev Sci Instr, v.81, 024701, 2010)

Three tone IMD



Saturation

Slope ≈1

Determining absolute IMD current

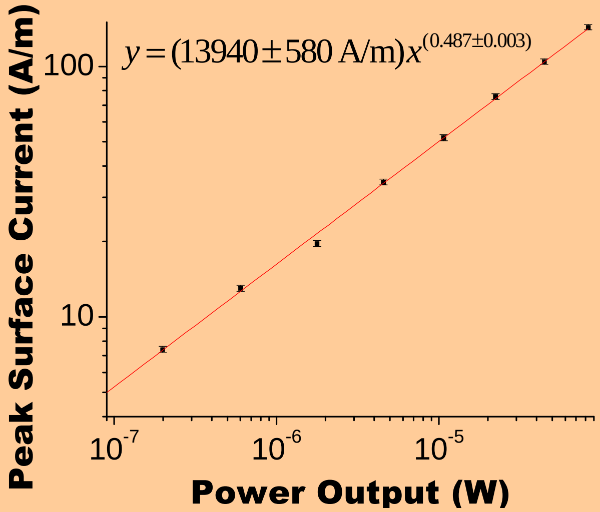
$$P_{diss}=P_{in}-P_{out}=R_S K_{peak}^2\cdot\overset{\text{From IE3D}}{\left[\frac{1}{2}\int\int_{Surface}\vec{K}\cdot\vec{K}\,dS\right]}\cdot\left[1+\frac{G}{R_S}f_d\tan\delta\right]$$

(Pease, at al,.Rev Sci Instr, v.81, 024701, 2010)

and also:

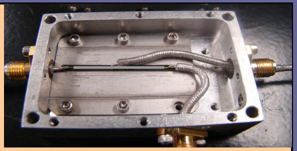
$$P_{diss}=P_{in}-P_{out}=4P_{Available}\frac{\left(\frac{Q_L}{Q_u}\right)\left(1-\frac{Q_L}{Q_u}\right)}{1+\frac{\beta_2}{\beta_1}}$$

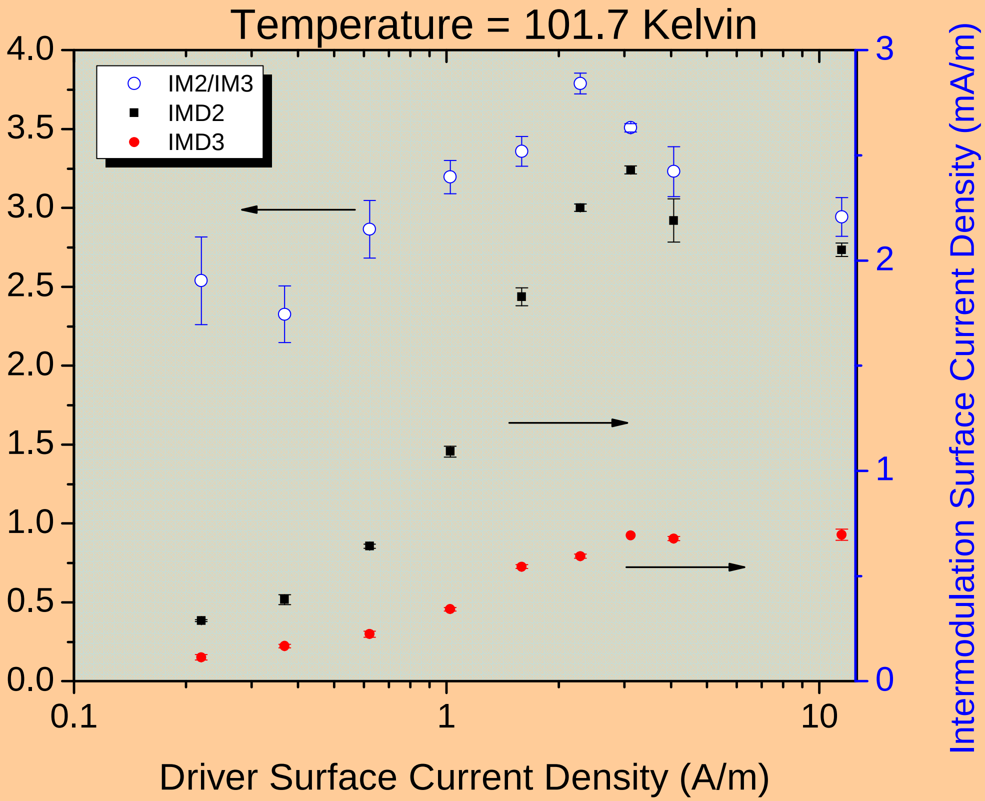
G, f_d	Geometrical factors
R_S	Surface resistance [Ω]
K	Surface current density [A/m]
β_1, β_2	Coupling coefficients
Q_L, Q_u	Loaded & unloaded Q factors



∴ Surface current density is known from the output power for any excitation in the resonant mode. (This includes IMD.)

Results: Power Sweep



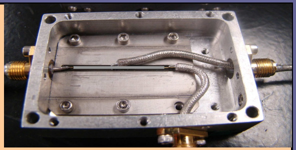


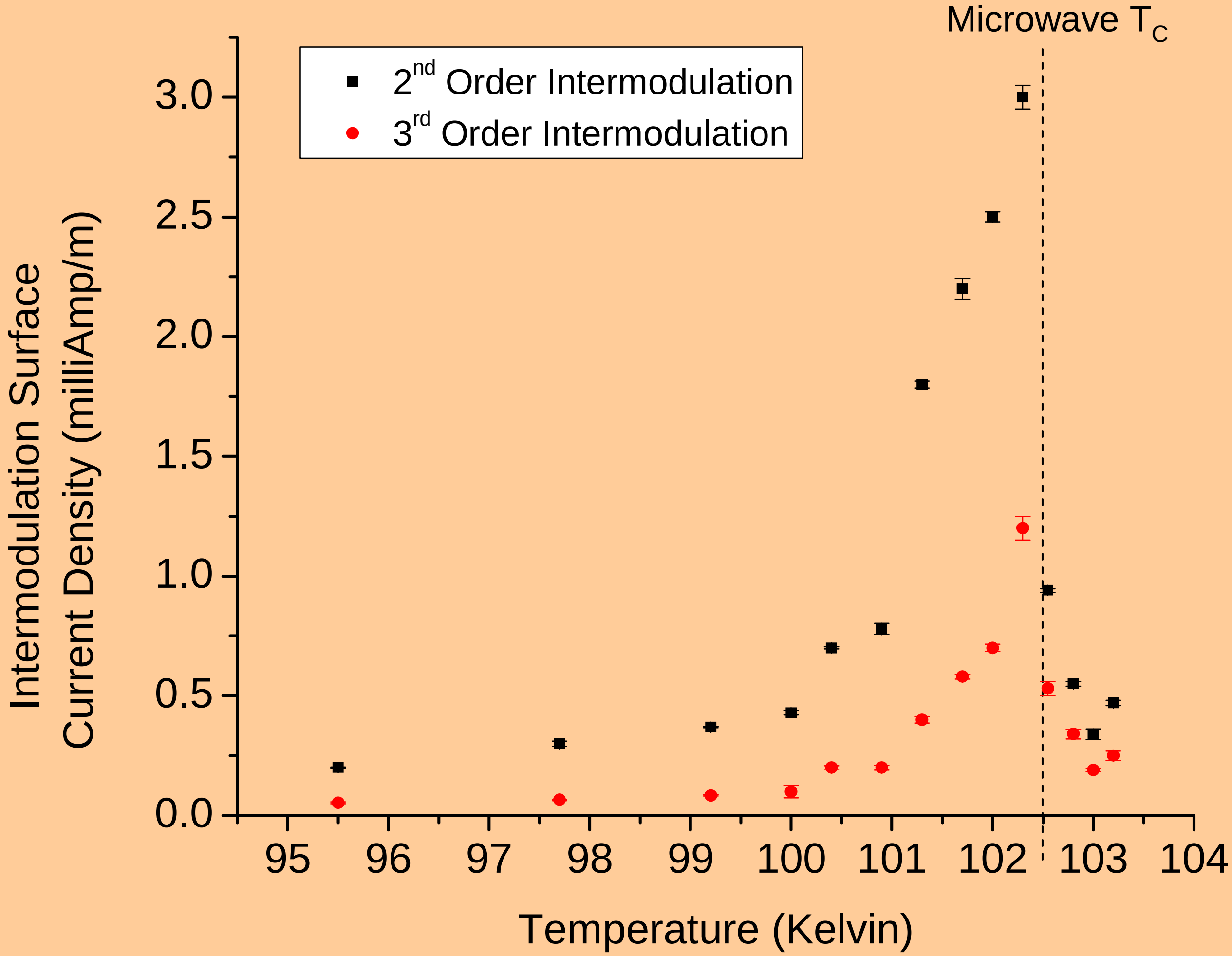
$$\frac{2^{nd}\text{ Order IMD Current}}{3^{rd}\text{ Order IMD Current}}$$

Spontaneous surface current that breaks TRS: $H_{TRSB} \propto \frac{H_{IMD2}}{H_{IMD3}}$

(S.C. Lee, et al., Phys Rev B, **vol 71**, 014507, 2005)

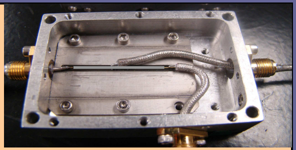
Results: Phase Transition

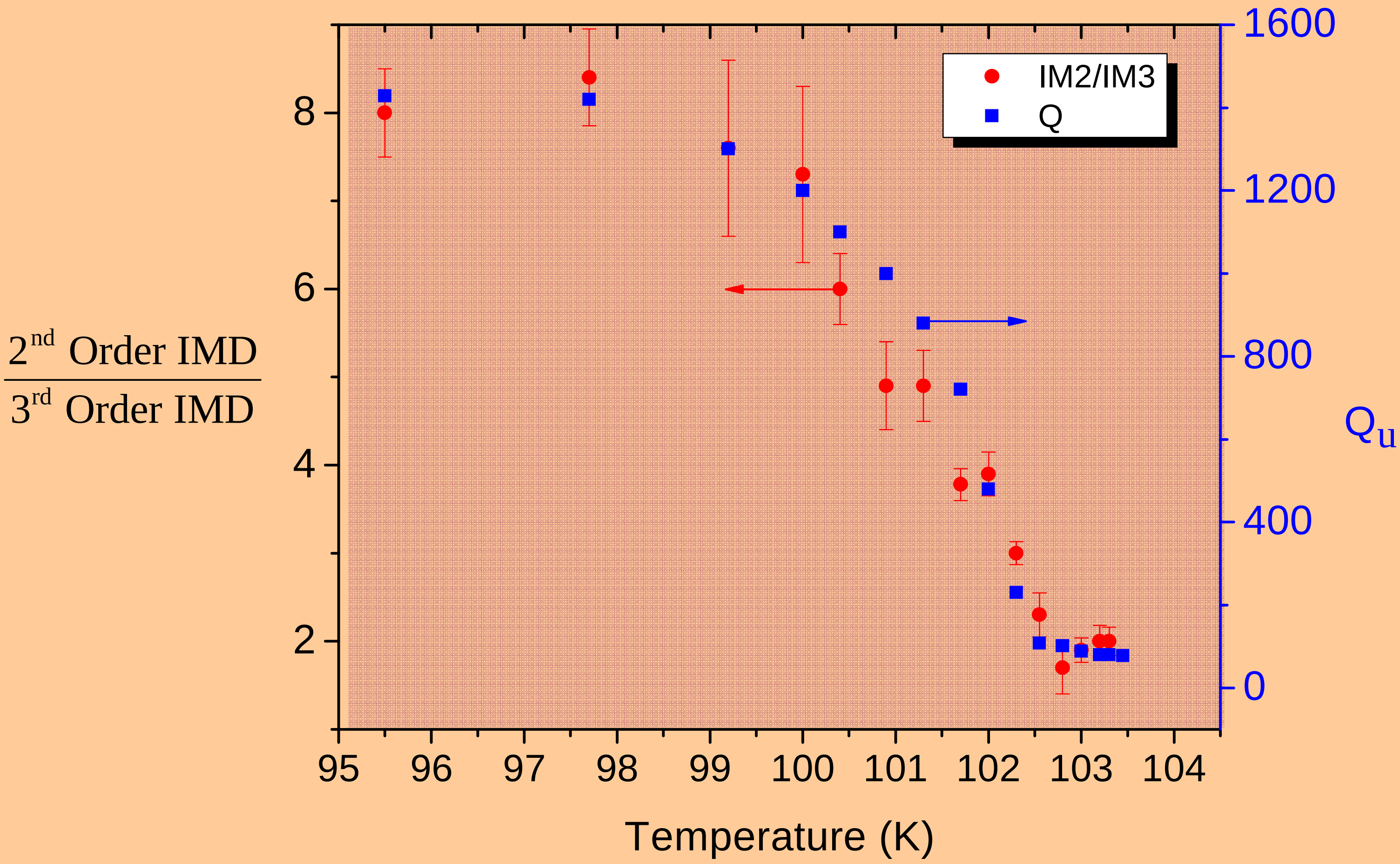




An explosion of both orders corresponds to the phase transition

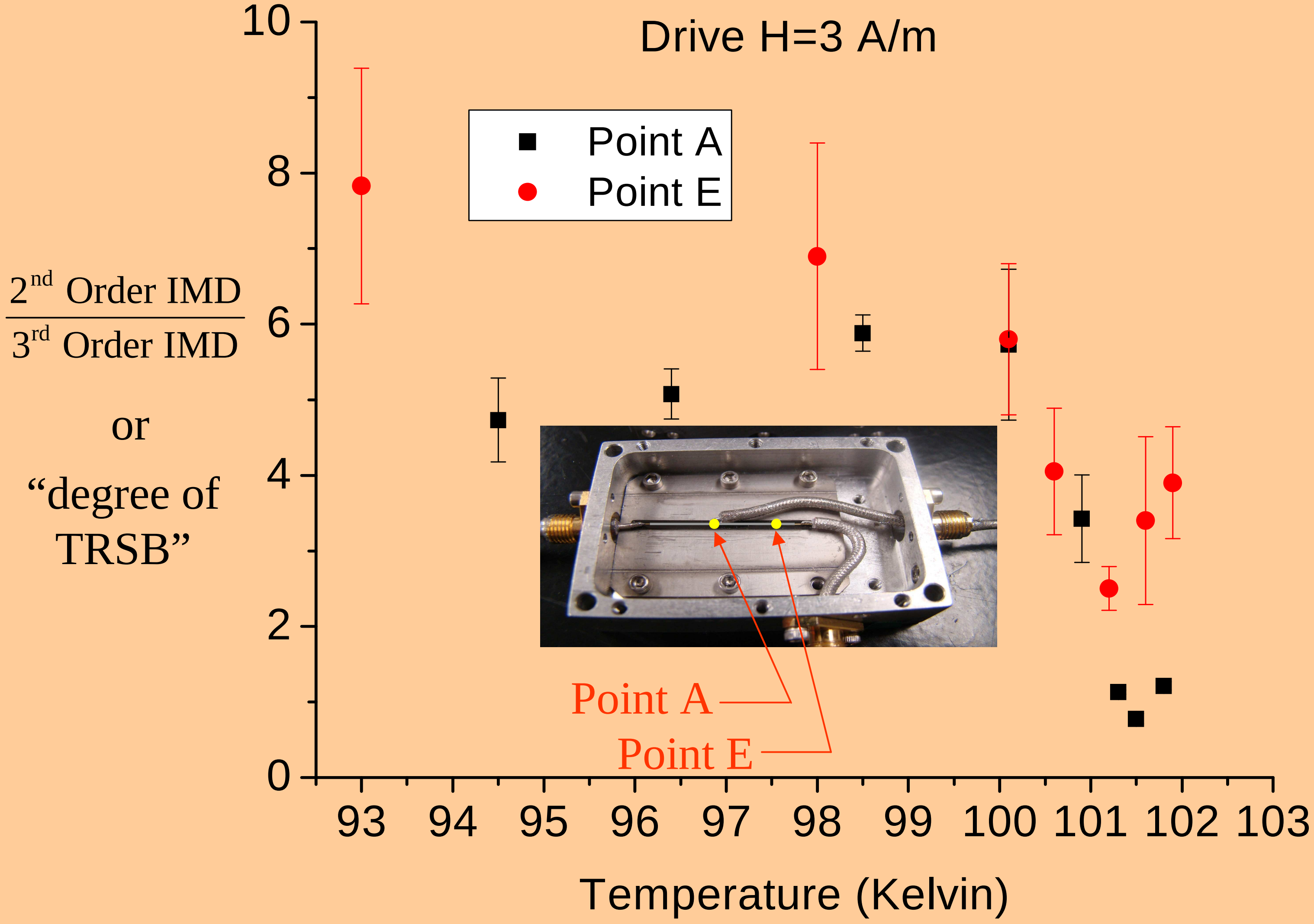
Results: TRSB





The degree of TRSB correlates with the resonator Q.

Results: The Potential to Scan Across a Sample



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WHERE DISCOVERIES BEGIN

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- Even and odd order nonlinearity can be measured at the same frequency using 3-tone intermodulation distortion.
- 3-tone IMD provides insight into the occurrence of spontaneous time reversal symmetry breaking currents in high temperature superconductor devices.
- Going forward this method will be used to map the TRSB nonlinearity throughout the superconductor.

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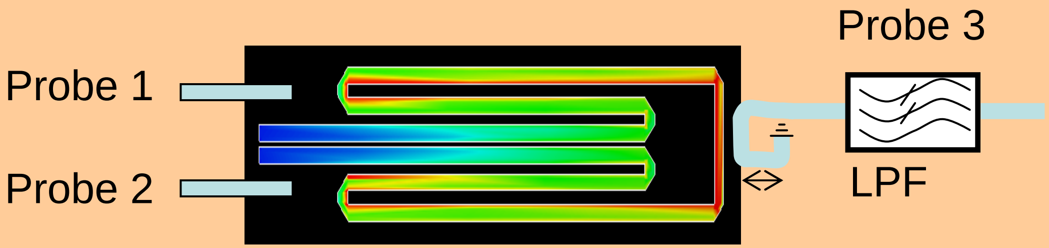
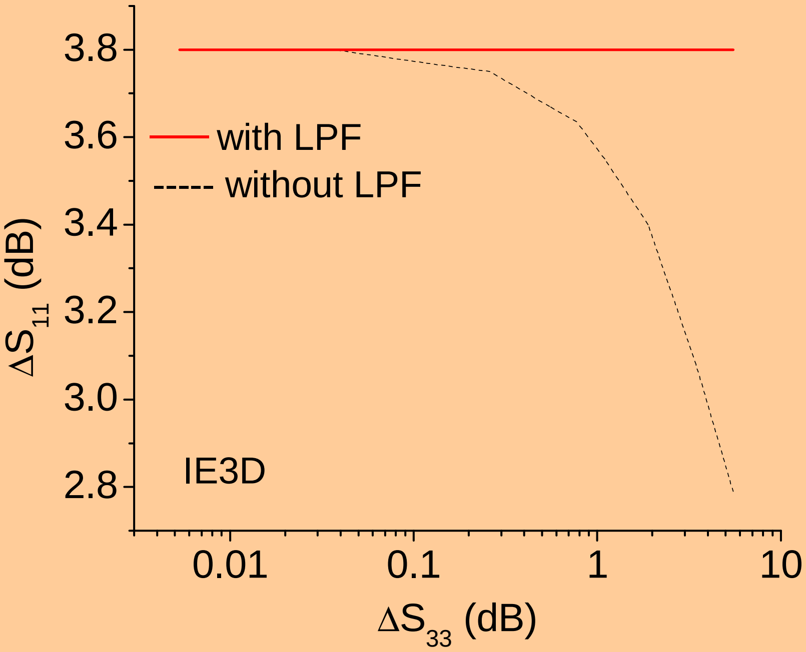
Appendix

Risky assumptions that needed to be scrutinized

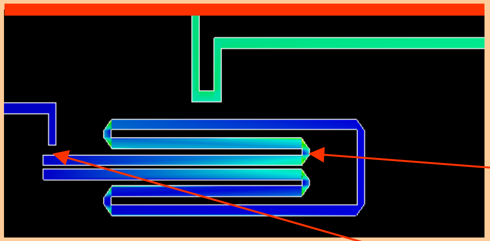
1. The coupling on the 3rd port is sufficiently weak that P_{diss} is readily known





2. The current distribution of the IMD signal is also T.E.M.

In-band IMD currents are generated locally by the probe signal and then excite the resonance:



E-wall →

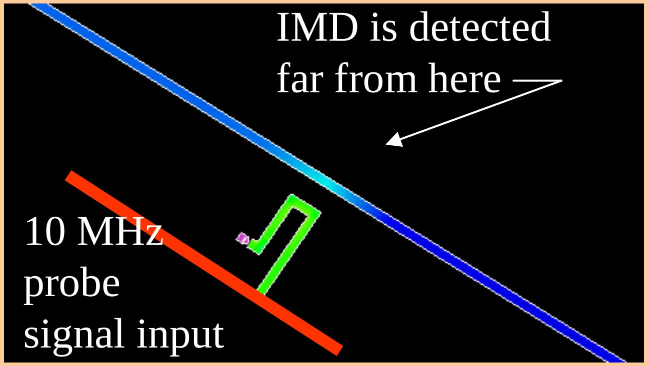
150 KHz probe signal input

IMD is generated here…

And detected here

“guitar resonator”

“Straight Microstrip Line”



IMD is detected far from here

10 MHz probe signal input

IE3D simulations show that current is induced locally by the out-of-band probing signal.

3. The surface current can be determined from the dissipated power.

$$P_{diss} = P_S + P_{diel}$$

$$P_{diss} = \underbrace{\frac{1}{2}\iint R_S \vec{H}\cdot\vec{H}^{*} dS}_{\text{also}} + \omega \mathrm{U}\cdot\left(\mathrm{f_d}\tan\delta\right)$$

$$Q_S = G / R_S = \omega U / P_S$$

$$P_{diss} = \frac{1}{2}\iint R_S \vec{H}\cdot\vec{H}^{*} dS + \frac{\mathrm{G}}{\mathrm{R_S}}\cdot\frac{1}{2}\iint R_S \vec{H}\cdot\vec{H}^{*} dS\cdot\left(f_d \tan\delta\right)$$

$$P_{diss} = \frac{1}{2}\iint R_S \vec{H}\cdot\vec{H}^{*} dS\cdot\left[1+\frac{G}{R_S}\cdot\left(f_d \tan\delta\right)\right]$$

$$P_{diss} = K_{\mathrm{max}}^{2} A\cdot R_S\cdot\left[1+\frac{G}{R_S}\cdot\left(f_d \tan\delta\right)\right]$$

Simulate with IE3D to find
A and K_{max}