Film Thickness Correction for the Surface Reactance and Surface Resistance Measured with a Sapphire Dielectric Resonator

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July 22, 2011, rev. 2

Your measurement of frequency shift, Δf, from the frequency at the lowest power, f_o, gives the "effective change in surface reactance," $\Delta X_{S,eff} = \omega\mu_o \cdot \Delta\lambda$, where $\lambda=\lambda_o+\Delta\lambda$ is the depth of penetration of magnetic field into the superconductor's surface. λ is not the London penetration depth, but the London effect is included in it. Other effects, in particular Josephson penetration at grain boundaries, contribute to λ as well. λ changes during the microwave measurements, which causes the "electromagnetic volume" of the resonator to change. This results in the frequency shift that you measure. $\omega=2\pi f$ is the angular frequency of the microwaves. Just use f_o when relating surface reactance to penetration depth. The percent difference between f_o and frequency at high field is negligible.

Because the film's thickness, *d*, is similar to that of λ, some of the RF energy penetrates through the film, and the substrate participates in the resonator volume. Also, the energy that penetrates the film either comes back into the sapphire, or is lost in the substrate. Thus, the measured surface resistance, $R_{S,eff}$, is affected by the thinness of the film. Two corrections need to be applied to R_S. One accounts for reduced dissipation when there is less film materials available. The other, R_{trans}, accounts for the energy that is lost in the substrate.

The corrections were given by Klein[1]

$$R_S = \frac{R_{S,eff} - R_{trans}}{f(d/\lambda)}$$ where the function, *f(d/λ)*, is

$$f(d/\lambda) = \coth(d/\lambda) + \frac{d/\lambda}{\sinh^2(d/\lambda)}$$ and

$$R_{trans} = \sqrt{\varepsilon_r}\,\frac{(2\pi f_o \mu_o \lambda)^2}{Z_o \sinh^2(d/\lambda)}$$ and

$$\Delta X_S = \Delta X_{S,eff}\tanh(d/\lambda)$$

ε_r is the permittivity of the substrate relative to the cavity. $Z_o=377\Omega/\sqrt{\varepsilon_{r,sapphire}}=122\ \Omega$ is the impedance of the sapphire. So lanthanum aluminate has $\varepsilon_r=23.6$. Our sapphire was found using the TE_{012} mode to have $\varepsilon_r=9.6$. So in the calculation of R_{trans} you will use 23.6/9.6=2.5 for the relative dielectric constant. Although these equations appear to have been developed for the case of an air filled cavity resonator, later writings by the same group apply them to sapphire dielectric resonators[2].

In order to figure out the corrected values, it is necessary to know λ. This cannot be known directly. We will use an iterative process combined with a value for λ at zero power taken from the literature. We use a value for λ at zero current and temperature[3] for $Tl_2Ba_2CaCu_2O_8$ thin films from the same manufacturer (DuPont) at 77K of λ_{ab}=200 nm. The subscript "ab" accounts for the anisotropy of the superconducting material. It is the penetration depth for the case that the film is c-axis oriented and the shielding current is in the ab-plane. This is the case in our dielectric resonator measurements. Using the empirical form for the temperature dependence[3,4]

$$\lambda(T) = \frac{\lambda(0)}{\sqrt{1-\left(\frac{T}{T_C}\right)^2}}$$

now understood to represent the d-wave dirty limit[5], this provides at, for example, $T/T_C=0.916$ a value of $\lambda_{ab}=500$ nm. We also need to recognize that there is some error in this value of $\lambda(T=0)$. Annealing the superconductors reduces the carrier density, which increases $\lambda(T=0)$, and we are not going to account for that just yet.

The iterative calculations

The measured $\Delta X_{S,eff}$ gives a penetration depth change, $\Delta\lambda_1=\Delta X_{S,eff}/\omega\mu_o$, which is wrong, since $\Delta X_{S,eff}$ is not the correct change in surface reactance. Nevertheless calculate $\Delta\lambda_1$ and add it to the original penetration depth to have $\lambda_o+\Delta\lambda_1$, which again is wrong. But now recompute the surface reactance change using this new penetration depth, $\Delta X_{S,2}=\Delta X_{S,eff}\cdot\tanh[d/(\lambda_o+\Delta\lambda_1)]$. Then compute a new penetration depth change, $\Delta\lambda_2=\Delta X_{S,2}/\omega\mu_o$ and add it to the original penetration depth to get the new penetration depth, $\lambda=\lambda_o+\Delta\lambda_2$. This really isn't the penetration depth because of the guess in the zero field value λ_o, but it will be used to estimate the true surface resistance fairly accurately. Compute the final surface reactance change, $\Delta X_S=\Delta X_{S,eff}\cdot\tanh[d/(\lambda_o+\Delta\lambda_2)]$ and once again find $\Delta\lambda_3$ from it. You could do more iterations, but by this point additional calculations of ΔX_S will not change the value by more than 1%. In all calculations, ω is equal to $2\pi f_o$.

Next you need to compute the corrected surface resistance. This is more straightforward since iteration is not necessary. Use the value for $\lambda_o+\Delta\lambda_3$ for λ in the above equations for R_S. The thickness of the film, I believe, is 400 nm. It might be as large as 650 nm people tell me, but I have RBS evidence that suggests 400 nm is right. So we will use d=400 nm.

Example

Suppose you calculate $\Delta X_{S,eff}=5$ mΩ and $R_S=1$ mΩ from your round robin at $f_o=5.556\times10^9$ Hz . Then using $\lambda_o=500$ nm and d=400 nm, add the following five columns to the spreadsheet to correct for the surface reactance:

$\Delta\lambda_1$	$\Delta X_{S,2}=\Delta X_{S,eff}\tanh[d/(\lambda_o+\Delta\lambda_1)]$	$\Delta\lambda_2=\Delta X_{S,2}/(\omega\mu_o)$	$\Delta X_{S,3}=\Delta X_{S,eff}\tanh[d/(\lambda_o+\Delta\lambda_2)]$	$\Delta\lambda_3$
115 nm	2.86 mΩ	65 nm	3.05 mΩ	70 nm

To correct for the surface resistance, add the following three columns to your spreadsheet:

$R_S = \dfrac{R_{S,eff}}{\coth\left(\dfrac{d}{\lambda_o + \Delta\lambda_3}\right) + \dfrac{d/(\lambda_o + \Delta\lambda_3)}{\sinh^2\left(\dfrac{d}{\lambda_o + \Delta\lambda_3}\right)}}$	$R_{trans} = (2.5x10^7\ \Omega/\mathrm{m}^2)\cdot\dfrac{(\lambda_o + \Delta\lambda_3)^2}{\sinh^2\left(\dfrac{d}{\lambda_o + \Delta\lambda_3}\right)}$	$R_{S,total}$
0.350 mΩ	0.014 mΩ	0.364 mΩ

Check these values by hand and make sure that you get them before adding them to your spreadsheet.

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