Experimental Observation and Control of Wave Dispersion

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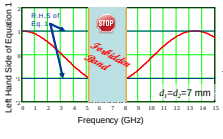
Dispersive Effects

- Electrons in a Lattice
- EM wave in a solid
- Sound in elastic media

Kronig-Penny potential in the Schrödinger equation has a transcendental solution:

$$\cos(k_1 d_1)\cos(k_2 d_2) - \frac{k_1^2 - k_2^2}{2k_1 k_2}\sin(k_1 d_1)\sin(k_2 d_2) = \cos(\beta(d_1 + d_2)) \quad (1)$$



Wave, $\beta = 2\pi/\lambda$

$k_i = \frac{\omega}{v_i} = \frac{2\pi}{\lambda_i}$ v_i = wave speed

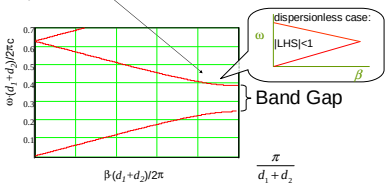
K.-P. potential

d_1 d_2 d_1+d_2=Lattice Constant

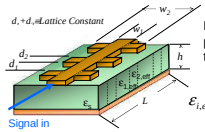
λ_i=wavelength in region "i"

$\frac{d\omega}{d\beta} \neq \text{Constant} \Rightarrow \text{Dispersion!}$

$\beta = \cos^{-1}\left(\frac{\text{L.H.S.}}{d_1 + d_2}\right)$



Periodic Transmission Lines



Periodic variation in ε_{eff} produces a periodic impedance mismatch of the wave.

$$\varepsilon_{i,eff} \approx \frac{\varepsilon_S + 1}{2} + \frac{\varepsilon_S - 1}{2\sqrt{1 + 12(h/w_i)}}$$

S_{11} ≡ Measured reflection coefficient magnitude and phase

S_{21} ≡ Measured transmission coefficient magnitude and phase

$$e^{j(\alpha+j\beta)L} = \frac{1 - S_{11}^2 + S_{21}^2 + \sqrt{(1 + S_{11}^2 - S_{21}^2)^2 - (2S_{11})^2}}{2S_{21}} \quad (2)$$

Propagation constant, solved by inverting this equation

Attenuation coefficient (Ref 1)

Experiment Outline

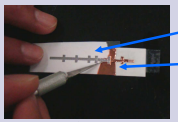
1. Write C-based code to evaluate Equation 1 and to invert Equation 2
2. Design a periodic transmission line using an EM field simulator
3. Fabricate the periodic transmission line
4. Measure the transmission and reflection coefficients vs. frequency
5. Use the computer program to compute β vs. frequency with Eq. 2
6. Plot the dispersion relation in the extended or reduced zone scheme
7. Attempt some "dispersion engineering" with an impurity

Condensed Abstract

- Dispersion: Deviations from the simple model of constant phase velocity
- Goal 1: Find a hands-on way to investigate the dispersion of electron waves in a crystal lattice
- Goal 2: Introduce defects into the crystal which result in controllable states in the band gap
- Process: Simulate crystals using hand-made transmission lines with periodic variations
- Analysis: Convert the transmission and reflection coefficients into band structure
- Purpose: Results in an experimental examination of the band theory of solids
- Findings: Defects produce engineered states in the band gap

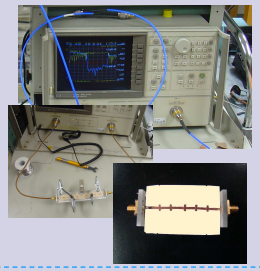
Experiment

The author uses IE3D to layout the structure and to perform method-of-moments EM simulation.

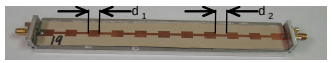


The dispersive structure is hand-fabricated using an *Exacto™* knife.

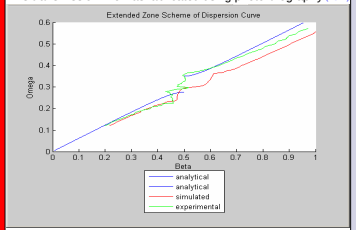
Transmission & reflection parameters of the dispersive structure are measured with a vector network analyzer.



Results



This transmission line was fabricated using photolithography.(Ref 2)



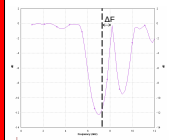
The dispersion curve generated using the above transmission line is generated three ways:

Analytic: Using Equation 1

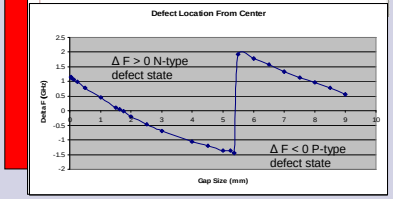
Simulated S-Parameters: Using Equation 2 and T&R coefficients from EM sim.

Measured: Using Equation 2 and T&R coefficients from measurement

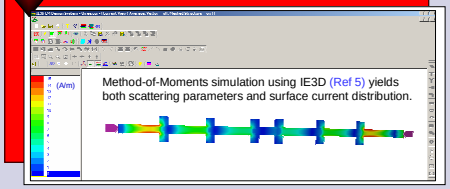
Distance of defect from the center of the band gap (Delta F)

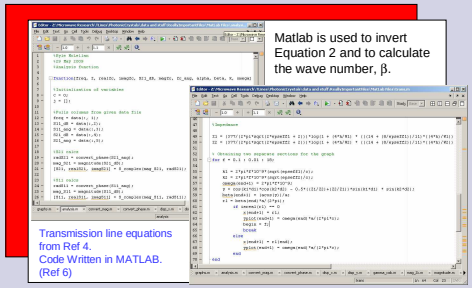


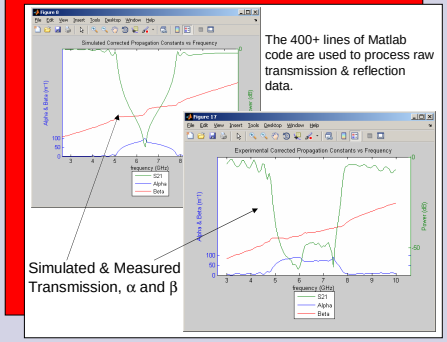
As the size of the gap 'G' increases, the state in the gap appears at a continually lower frequency until it reaches the critical point, where the defect jumps back to the far right of the band gap.

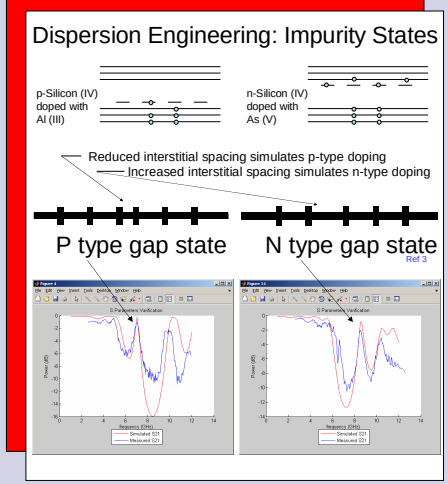


Analysis and Modeling









References

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2. C. Isaac Angert and S.K. Remillard, "Dispersion in One-Dimensional Photonic Band Gap Periodic Transmission Lines," Microwave and Optical Technology Letters, **51**, no. 4, 1010-1013 (2009).
3. E. Yablonovitch, et. al, "Donor and Acceptor Modes in Photonic Band Structure," Phys. Rev. Lett., **67**, no. 24, 3380-3383 (1991).
4. Brian C Wadell, Transmission Line Design Handbook, Artech House, Inc, Norwood, MA, 1991, Page 94.
5. IE3D EM Design System, Zeland Software Inc, Fremont, CA.
6. MATLAB, The MathWorks, Natick, MA.

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